Contents

1 Number and Algebra

1A: Sequences and Series

Useful notes

A sequence is a list of numbers arranged in order.

Arithmetic Progressions (AP)

An arithmetic sequence or arithmetic progression (AP) is a special sequence. In an AP, the difference between any two consecutive terms is constant.

This difference denoted by 'd' is called the 'common difference'.

The first term of an AP is given by u_1.

In general, for an AP, the common difference is given by,

$d = u_{n+1} - u_n$.

The terms of an AP are given by

$u_1, u_1 + d, u_1 + 2d, u_1 + 3d, \ldots$

This gives us the general term, that is the n^{th} term of an AP as,

$u_n = u_1 + (n-1)d$.

The sum to 'n' terms of an AP is given by,

$S_n = \frac{n}{2}(u_1 + u_n)$ or $S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right]$, where u_1 refers to the last term.

In general for any sequence,

$u_n = S_n - S_{n-1}$

Geometric Progressions (GP)

A geometric sequence or geometric progression (GP) is a special sequence. In a GP, the ratio of any two consecutive terms is constant.

This ratio denoted by 'r' is called the 'common ratio'.

The first term of a GP is given by u_1.

In general, for a GP, the common ratio is given by,

$r = \frac{u_{n+1}}{u_n}$.

Exercise

1 *(no calculator allowed)*
Consider the sequence 5, 11, 17, 23, 29, …
Assuming that the pattern continues,

(a) verify that the sequence is an arithmetic progression.

(b) write down the first term and the common difference.

(c) write down the formula for u_n, the n^{th} term.

(d) find the 15^{th} and 20^{th} term of the sequence.

(e) find 'n' given that $u_n = 71$.

(f) check whether 123 is a term of this sequence.

2 *(no calculator allowed)*
Consider the sequence 51, 47, 43, 39, 35, …
Assuming that the pattern continues,

(a) verify that the sequence is an arithmetic progression.

(b) write down the first term and the common difference.

(c) write down the formula for u_n, the n^{th} term.

(d) find the 12^{th} and 16^{th} term of the sequence.

(e) find 'n' given that $u_n = 25$.

(f) check whether -30 is a term of this sequence.

3 *(no calculator allowed)*
The general term of a sequence is defined by $u_n = 3n - 4$.

(a) Show that this sequence is an arithmetic progression.

(b) Find the first term and common difference.

(c) Find the first term of the sequence that is greater than 345.

4 *(no calculator allowed)*
The general term of a sequence is defined by $u_n = \dfrac{42 - 4n}{3}$.

(a) Show that this sequence is an arithmetic progression.

(b) Find the first term and common difference.

(c) Find the first term of the sequence that is negative.

5 *(no calculator allowed)*
The general term of a sequence is defined by $u_n = n^2 - 3n$. Is this sequence an arithmetic progression? Explain your answer.

6 *(no calculator allowed)*
The general term of a sequence is defined by $u_n = 0.5^n$. Is this sequence an arithmetic progression? Explain your answer.

7 *(no calculator allowed)*
Three consecutive terms of an arithmetic progression are $6q$, $2q - 2$ and -8. Find the value of q.

8 *(no calculator allowed)*
Three consecutive terms of an arithmetic progression are $3k + 4$, $k + 3$ and 0. Find the value of k.

9 *(no calculator allowed)*
Three consecutive terms of an arithmetic progression are given by $p^2 - 6$, $p + 2$ and 7. Find the value of p.

10 *(no calculator allowed)*
The 6^{th} and 11^{th} terms of an arithmetic progression are 40 and 75 respectively. Find

(a) the first term and common difference,

(b) the general term,

(c) the 19^{th} term.

11 *(no calculator allowed)*
The 8^{th} and 20^{th} terms of an arithmetic progression are 7 and -41 respectively. Find

(a) the first term and common difference,

(b) the general term,

(c) the first negative term.

12 *(no calculator allowed)*
Five numbers in an arithmetic progression are such that the first, and the last terms are 5 and 18 respectively. Find

(a) the common difference,

(b) the remaining terms.

13 *(no calculator allowed)*
An arithmetic progression has terms –4, 1, ..., 76. Find

(a) the common difference,

(b) the number of terms in the arithmetic progression.

14 *(no calculator allowed)*
How many two-digit positive integers are divisible by 6?

15 *(no calculator allowed)*
How many three-digit positive integers are divisible by 5?

16 *(no calculator allowed)*
The sum of the 2^{nd} and 4^{th} terms of an arithmetic progression is 14. The sum of the 5^{th} and 7^{th} terms of the same arithmetic progression is 26. Find

(a) the first term and the common difference,

(b) the sum of the 3^{rd} and 10^{th} terms.

17 *(no calculator allowed)*
In an arithmetic progression, the ratio of the 6^{th} term to the 8^{th} term is $\frac{4}{5}$. Find the ratio of the 4^{th} term to the 7^{th} term.

18 *(calculator allowed)*
Find the following sums to the given number of terms.

(a) 4 + 8 + 12 + 16 + … to 25 terms

(b) 15 + 11 + 7 + 3 + … to 30 terms

(c) −1 − 1.5 − 2 − 2.5 + … to 22 terms

(d) $\frac{1}{3}+\frac{2}{3}+1+1\frac{1}{3}+\ldots$ to 12 terms

19 *(calculator allowed)*
Find the following sums.

(a) 5 + 9 + 13 + … + 93

(b) 134 + 127 + 120 + … − 41

(c) 1 + 1.2 + 1.4 + … + 5

(d) $-5-4\frac{3}{4}-4\frac{1}{2}-\ldots-0$

20 *(calculator allowed)*
Find the sum of the arithmetic series of 40 terms whose first term is 5 and common difference is 2.

21 *(calculator allowed)*
An arithmetic series has 15 terms. Its first term is –2 and last term is 50. Find its sum.

22 *(calculator allowed)*
Find the sum of all even numbers between 21 and 40, inclusive.

23 *(calculator allowed)*
Find the sum of all multiples of 12 between 1 and 100, inclusive.

24 *(calculator allowed)*
The third term of an arithmetic series is 10. The sum of its first 50 terms is 3875. Find

(a) its first term and common difference,

(b) its 7^{th} term,

(c) the sum of its first 32 terms.

25 *(calculator allowed)*
The fourth term of an arithmetic series is 15. The sum of its first 22 terms is 1485. Find

(a) its first term and common difference,

(b) its 10^{th} term,

(c) the sum of its first 25 terms.

26 *(calculator allowed)*
The sum of 5 consecutive terms of an arithmetic series is 40. The sum of its first 20 terms is 1210. Find

(a) its first term and common difference,

(b) its 5^{th} term,

(c) the sum of its first 24 terms.

27 *(calculator allowed)*
Three positive numbers are consecutive terms of an arithmetic progression. Find the three numbers if their sum is 30 and product is 750.