**Non-repeated quadratic factor:**

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

## Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q = 2 \sin \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$

$$\sin P - \sin Q = 2 \cos \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$

$$\cos P + \cos Q = 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$$

$$\cos P - \cos Q = -2 \sin \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$$

**Principal values:**

$$-\frac{1}{2}\pi \le \sin^{-1} x \le \frac{1}{2}\pi \qquad (|x| \le 1)$$

$$0 \le \cos^{-1} x \le \pi \quad (|x| \le 1)$$

$$-\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

## Derivatives

| $\mathrm{f}(x)$ | $\mathrm{f}'(x)$ |
|---|---|
| $\sin^{-1} x$ | $\dfrac{1}{\sqrt{1-x^2}}$ |
| $\cos^{-1} x$ | $-\dfrac{1}{\sqrt{1-x^2}}$ |

| | |
|---|---|
| $\tan^{-1} x$ | $\dfrac{1}{1+x^2}$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\sec x$ | $\sec x \tan x$ |

## Integrals *(Arbitrary constants are omitted; a denotes a positive constant.)*

| $f(x)$ | $\int f(x)\,dx$ | |
|---|---|---|
| $\dfrac{1}{x^2+a^2}$ | $\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$ | |
| $\dfrac{1}{\sqrt{a^2-x^2}}$ | $\sin^{-1}\left(\dfrac{x}{a}\right)$ | $(\lvert x\rvert < a)$ |
| $\dfrac{1}{x^2-a^2}$ | $\dfrac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right)$ | $(x > a)$ |
| $\dfrac{1}{a^2-x^2}$ | $\dfrac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right)$ | $(\lvert x\rvert < a)$ |
| $\tan x$ | $\ln(\sec x)$ | $(\lvert x\rvert < \tfrac{1}{2}\pi)$ |
| $\cot x$ | $\ln(\sin x)$ | $(0 < x < \pi)$ |
| $\operatorname{cosec} x$ | $-\ln(\operatorname{cosec} x + \cot x)$ | $(0 < x < \pi)$ |
| $\sec x$ | $\ln(\sec x + \tan x)$ | $\left(\lvert x\rvert < \dfrac{1}{2}\pi\right)$ |

## Vectors

The point dividing $AB$ in the ratio $\lambda : \mu$ has position vector $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

**Vector product:**

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

## Finding Inverse Function

Set $y = f(x)$ and make $x$ the subject.

### *Notes when finding inverse*

- If a function is a quadratic equation in terms of $x$, either use completing the square or apply the formula for finding roots.
- When you apply square root in this case, remember to add $\pm$. Choose the sign by looking at the domain of f
- $D_{f^{-1}} = R_f, R_{f^{-1}} = D_f$

**Example**

The function f is defined by $f : x \mapsto (x-3)^2 + 1, \quad x \le 3$.

Show that the inverse function of f exists and find $f^{-1}$ in similar form.

***Solution:***

Since every horizontal line $y = k, k \ge 1$ cuts the graph $y = f(x)$ exactly once, f is 1-1 and $f^{-1}$ exists.

Let $y = (x-3)^2 + 1$

$(x-3)^2 = y - 1$

$\Rightarrow x = 3 \pm \sqrt{y-1}$

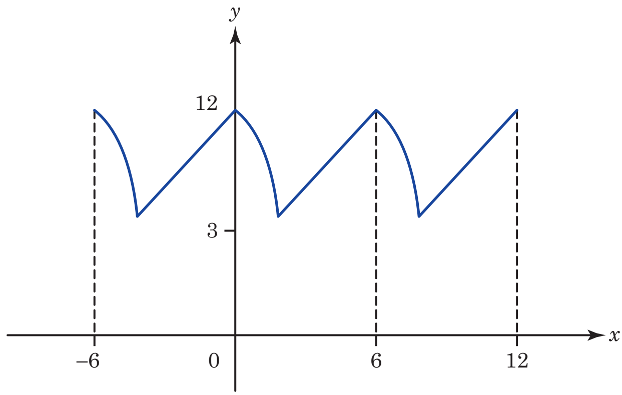
Since $x \le 3, x = 3 - \sqrt{y-1}$

$f^{-1} : x \mapsto 3 - \sqrt{x-1}, x \ge 1.$

### *Notes*

- $y = f^{-1}(x)$, $y = f(x)$ intersects $y = x$ at the same point.
- $f^{-1}f(x) = x$ and $ff^{-1}(x) = x$ by definition. There is no need to prove this. However, the two functions may be different if their domains are different.

**(ii)**




**Example**

The function f is defined by $f : x \to \dfrac{2-3x}{3-x}, x > 3$.

Define, in a similar manner, the inverse function $f^{-1}$ and show that $f^2(x) = x$.

Hence determine $f^{13}$ in a similar manner.

***Solution:***

$$f(x) = \frac{2-3x}{3-x}, \quad x > 3,$$

$$\text{Let } y = \frac{2-3x}{3-x}$$

$$(3-x)y = 2-3x$$

$$(3-y)x = 2-3y$$

$$x = \frac{2-3y}{3-y}$$

$$\therefore f^{-1} : x \to \frac{2-3x}{3-x}, x > 3$$

$$f^2(x) = f[f(x)]$$

$$= f[f^{-1}(x)] \quad (\because f = f^{-1} \text{ and } ff^{-1} = f^{-1}f = x)$$

$$= x$$

$$f^{13}(x) = f[f^{12}(x)] = f(x)$$

$$= \frac{2-3x}{3-x}$$

***Note***

Recognise the pattern generated.

## Polarity of denominator unknown

When the polarity of denominator of a rational function is unknown, multiply throughout the inequality by the lowest even power of the denominator so that the direction of the inequality sign will not be affected.

**Example**

Solve the inequality $\dfrac{(x-1)(x+2)}{(3x+4)} \le 0$

***Solution:***

$$\frac{(x-1)(x+2)}{(3x+4)} \le 0, \quad x \ne -\frac{4}{3}$$

Multiplying throughout by $(3x+4)^2$,

$$\Rightarrow \quad \frac{(3x+4)^2(x-1)(x+2)}{(3x+4)} \le (3x+4)^2 \cdot 0$$

$$\Rightarrow \quad (3x+4)(x-1)(x+2) \le 0$$

$$\Rightarrow \quad x \le -2 \quad \text{or} \quad -\frac{4}{3} < x \le 1$$